Finish reading Chapter 16, pages 325 to 331 of Chapter 11, and Chapter 13 in the OpenCV book. Project 1 is due on Tuesday 23 October 2018.
The Midterm Exam will be held on Thursday 25 October during class time. The Exam will cover the material and reading assigned in homeworks 1, 2, and 3, and class lecture notes. The Midterm Exam will be closed book and closed notes. Only one 8.5 inch by 11 inch page of hand-written notes is allowed. No calculators and no other devices are allowed, except for a pencil, pen, eraser, and ruler.

**PROBLEM 3.1:**
(3 points) Consider a homomorphic processing system whose input-output relationship is given below:

\[ y(n_1, n_2) = \exp \left( \log(x(n_1, n_2)) \ast * h_{hp}(n_1, n_2) \right) \]

where \( h_{hp}(n_1, n_2) \) is the impulse response of a high-pass filter, \( x(n_1, n_2) \) is the input image, and \( y(n_1, n_2) \) is the corresponding output image.

(a) (1.5 points) Let \( x_2(n_1, n_2) = 2x(n_1, n_2) \), and let \( y_2(n_1, n_2) \) be the output of the homomorphic processing system when the input is \( x_2(n_1, n_2) \). Express \( y_2(n_1, n_2) \) in terms of \( y(n_1, n_2) \).

(b) (1.5 points) Let \( x_3(n_1, n_2) = (x(n_1, n_2))^2 \), and let \( y_3(n_1, n_2) \) be the output of the homomorphic processing system when the input is \( x_3(n_1, n_2) \). Express \( y_3(n_1, n_2) \) in terms of \( y(n_1, n_2) \).
PROBLEM 3.2:
(5 points) Consider an image consisting of $4 \times 4$ pixels with intensities as shown below.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

We wish to modify the gray scale of this image such that the processed image has a histogram that approximates the following desired histogram:

(a) (2 points) Following the procedure presented in class, determine a mapping that will achieve the desired objective.

(b) (1.5 points) Determine the processed image based on the mapping constructed in (a).

(c) (1.5 points) Draw the histogram of the processed image.
PROBLEM 3.3:
(4 points) Consider the subimage \( x(n_1, n_2) \) shown below. Assume zero values outside the borders of the shown subimage \( x(n_1, n_2) \).

\[
\begin{array}{cccc}
110 & 112 & 200 & 116 \\
0 & 110 & 108 & 108 \\
128 & 130 & 122 & 123 \\
135 & 10 & 125 & 225 \\
\end{array}
\]

(a) (2 points) Indicate below the values of the output \( y_1(n_1, n_2) \) of a 2D \( 3 \times 3 \) median filter applied to \( x(n_1, n_2) \).

\[
\begin{array}{cccc}
\text{ } & \text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } & \text{ } \\
\end{array}
\]

(b) (2 points) Indicate below the values of the output \( y_1(n_1, n_2) \) of a 2D \( 3 \times 3 \) rank order filter with rank 3 applied to \( x(n_1, n_2) \).

\[
\begin{array}{cccc}
\text{ } & \text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } & \text{ } \\
\end{array}
\]