Image Enhancement and Image Restoration

• Image Enhancement
  ➢ Objective: accentuate or improve appearance of features, for subsequent analysis or display (possibly, but not necessarily degraded by some phenomenon).
  ➢ Examples of features: edges, boundaries, dynamic range and contrast.
  ➢ Examples of applications:
    ✓ TV: enhance image for viewer (image quality, intelligibility, visual appearance).
    ✓ Preprocessing for machine identification.
  ➢ Enhancement is not necessarily needed because of degradation but can be used possibly to remove degradation.
Image enhancement and Image Restoration

- **Blurred or faint edges**
  - Sharp edges

- **Low contrast or dynamic range**
  - Modify low dynamic range

- **Noise**
  - Remove noise
Image Enhancement and Image Restoration

- Image Restoration
  - Objective: Removal or reduction of known degradations in an image
    - Examples of degradations: blurred image, where blurring caused by known phenomenon; known noise properties; degradation and geometric distortion and nonlinearities due to sensor or environment.

  ![Filter of camera](image1)  ![Low resolution or blurring due to camera or camera motion](image2)  ![Increase spatial resolution or sharpen](image3)  ![Inverse process](image4)
Image Enhancement and Image Restoration

- Image Restoration

Distortion is due to known process.

- More knowledge then enhancement and this knowledge is exploited to correct degradation.

- Objective is to make restored image resemble the original image.
Image Enhancement

- **Objective:** Make processed image better in some sense than unprocessed image ⇒ ideal desired image not well defined (not known) and depends on problem context.
  - **Examples:**
    - Images in space have often floating dust ⇒ original image has dust in it which appears as a noise ⇒ removing this “noise” is enhancement, not restoration.
    - An original undegraded image cannot be further restored but can be enhanced.
- **Main difficulty:**
  - Often difficult to quantify the criteria for enhancement.
  - Objective of image enhancement is dependent on the application context (application-dependent) ⇒ the criteria for enhancement are often subjective or too complex to be easily converted to useful objective measures ⇒ image enhancement algorithms tend to be qualitative and ad hoc.
Image Enhancement

• Types of distortions to correct:
  - Blur (Blurring due to camera motion, defocusing, …)
  - Noise (assumed to be additive often for simplicity, although not necessarily the case).
  - Contrast
  - Examples:
    - Space photography
    - Underwater photography
    - Film grain noise
Image Enhancement

• Basic Tools to achieve objective:
  ➢ Contrast and dynamic range modification
  ➢ Noise smoothing
  ➢ Edge detection
  ➢ Image interpolation (motion estimation)
  ➢ Pseudocoloring
Contrast and Dynamic Range Modification

- Contrast stretching
  - Degradation is commonly due to poor lighting.
  - Image probability distribution function (pdf) has narrow peak ⇒ poor contrast.

- Image intensities are clustered in a small region ⇒ available dynamic range is not very well utilized.
Contrast and Dynamic Range Modification

- Possible solution: increase overall dynamic range
  - ⇒ resulting image would appear to have a greater contrast
  - ⇒ expand the amplitudes from $a$ to $b$ to cover available intensity range.

\[
p(x_{\text{new}})
\]
Contrast and Dynamic Range Modification

- **Idea:** Gray scale or intensity level of an input image $x(n_1, n_2)$ is modified according to a specific transformation (function) $f(\cdot)$.

- **Note:** $f(\cdot)$ is usually constrained to be a monotonically non-decreasing function of $x \Rightarrow$ ensures that a pixel with higher intensity than another will not become a pixel with a lower intensity in output image $x_{\text{new}}$.

- Typical stretching operator:

$$
x_{\text{new}} = \begin{cases} 
\alpha x, & 0 \leq x \leq a \\
\beta(x - a) + \alpha a, & a \leq x \leq b \\
\gamma(x - b) + \beta(b - a) + \alpha a, & b \leq x \leq L
\end{cases}
$$
Contrast and Dynamic Range Modification

✓ Specific desired transformation depends on the application
  Example: compensation of display non-linearity ⇒ most suitable transformation depends on display non-linearity.

✓ In most applications, a good or suitable transformation can be identified by computing and analyzing the histogram of the input image to be enhanced.
  – The histogram is a scaled version of the image pdf.
  – The histogram gives pdf when scaled by the total number of pixels in the image.
Histogram Modification and Equalization

- **Definition:** The histogram of an image \( h(x) \) represents the number of pixels that have a specific intensity \( x \Rightarrow \text{number of pixels as a function of intensity } x \).

\[
h(x) = \text{scaled version of pdf } p(x)
\]

\[
\text{pdf } p(x) \approx \frac{h(x)}{\text{Total number of pixels in image}} = h_n(x)
\]

Normalization ensures that

\[
\sum_{x=0}^{L} h_n(x) = 1, 0 \leq x \leq L
\]
Histogram Modification and Equalization

- Remarks:
  - Histogram modification methods popular because computing and modifying histogram of an image requires little computations.
  - Experienced person can easily determine needed transformation by analyzing histogram characteristics. But if too many images ⇒ automatic method is desired.
  - For typical natural images, the desired histogram has a maximum around the middle of the dynamic range and decreases slowly as the intensity increases or decreases.

- Problem: Determine $f(\cdot)$ such that $h_{output}(x_{new}) = h_d(x_{new})$
Histogram Modification and Equalization

- **Histogram equalization**: special case of histogram modification where

  \[ h_d(x) = \text{constant} \]

  Redistribute pixels by assigning pixels uniformly to the given levels.

\[
\text{constant} = \frac{\text{Total number of pixels in image}}{\text{Number of intensity levels} (L_{\text{max}} - L_{\text{min}} + 1) \text{ in dynamic range}}
\]

- **For a 256×256 image, with 256 intensity levels:**

  \[
  \text{const} = \frac{(256)^2}{256} = 256 \quad \text{pixels assigned to each level}
  \]

- **How can we do assignment?**
Histogram Modification and Equalization

- **Cumulative method for histogram equalization**
  - Histogram equalization \(\Rightarrow\) desired histogram is constant at all levels.
  - **Problem**: Find transformation \(x_o = f(x_i)\) such that that \(h_{output}(x_o) = \text{const}\)

\[
\text{const} = \frac{\sum_{i=x_{\text{min}}}^{x_{\text{max}}} h(x_i)}{L} = \frac{\text{Total number of pixels}}{\text{Number of levels}}
\]

If normalized histogram \(\Rightarrow\) \(\sum_{i=x_{\text{min}}}^{x_{\text{max}}} h(x_i) = 1 \Rightarrow \text{const} = \frac{1}{L} \Rightarrow \text{uniform distribution}\)
Histogram Modification and Equalization

Solution:

✓ Compute input pdf

\[ p_i(x) = \frac{h_i(x)}{\sum_{x=x_{\min}}^x h_i(x)} = \text{normalized histogram} \]

✓ Choose

\[ f(x_i) = F(x_i) = \sum_{x=0 \text{ or } x_{\min}}^{x_i} p_i(x)dx = p(x \leq x_i) = \text{cumulative probability distribution of } x_i \]

+ scaling needed

✓ Why?

if \[ y = F(x_i) = \int_0^{x_i} p_i(x_i)dx_i \Rightarrow y \text{ uniformly distributed between } (0,1) \]

⇒ histogram uniformly distributed

⇒ need also to scale \( y \) because \( y \in (0,1) \) instead of \((0,L-1)\) or \((L_{\min},L_{\max})\)
Histogram Modification and Equalization

✓ **Note:** if $y = F(x_i) = \int_0^{x_i} p_i(x_i)dx_i \Rightarrow y$ uniformly distributed between (0,1)

**Proof:**

$\text{Prob}[y \leq a] = \text{Prob}[x_i \leq F^{-1}(a)] = F(F^{-1}(a)) = a$

where $0 \leq a \leq 1 \Rightarrow y$ is uniformly distributed
Histogram Modification and Equalization

✓ Since $x_i$ is a discrete variable, integral is replaced by summation:

$$y = \sum_{x=x_{\text{min}}}^{x_i} p_i(x)$$
only approximately uniformly distributed (because of discretization)

$\Rightarrow y_{\text{min}}$ not necessarily 0 since $y_{\text{min}} = p(x_i \leq x_{\text{min}}) = p_i(x_{\text{min}})$

✓ Scaling can be done as follows

$$y = \sum_{x=x_{\text{min}}}^{x_i} p_i(x)$$

$$x_o = \text{Round} \left[ \frac{y - y_{\text{min}}}{1 - y_{\text{min}}} (L_{\text{max}} - L_{\text{min}}) + L_{\text{min}} \right]$$

$$y = y_{\text{min}} \Rightarrow x_o = L_{\text{min}}$$

$$y = 1 \Rightarrow x_o = L_{\text{max}}$$
Histogram Modification and Equalization

- **Procedure:** $x_k, k=0, \ldots, L-1 =$ input amplitude levels
  $y_k, k=0, \ldots, L-1 =$ output amplitude levels

1. Compute the histogram of the image to be improved.
2. Normalize histogram;
   - Normalize amplitudes so that the sum of all values is equal to one and you have a pdf, $p_i(\cdot)$.
   - $0 \leq x_k \leq 1$ – rescale input amplitude
3. Compute
   $$y_k = R\left(\sum_{l=0}^{k} p_i(x_l)\right) \quad \text{where} \quad R\{\cdot\} \text{ - round to next level}$$
4. Move bins in $x_k$ to locations in $y_k$.
5. Scale $y_k$ to desired amplitude range (linear mapping).

**Note:** Resulting images have more contrast but appear somewhat unnatural $\Rightarrow$ better use non-uniform $h_d(x)$. 
Histogram Modification and Equalization

- **Example:**

\[ y_0 = R\{p_i(x_0)\} = R\{0.14\} \approx \frac{1}{7} \]

\[ y_1 = R\left\{\sum_{l=0}^{1} p_i(x_l)\right\} = R\{0.14 + 0.25\} = R\{0.39\} \approx \frac{3}{7} \]

\[ y_2 = R\{0.39 + 0.2\} = R\{0.59\} \approx \frac{5}{7} \]

\[ y_3 = R\{0.59 + 0.15\} = R\{0.74\} \approx \frac{6}{7} \]

\[ y_4 = R\{0.74 + 0.1\} = R\{0.84\} \approx \frac{6}{7} \]

\[ y_5 = R\{0.84 + 0.07\} = R\{0.91\} \approx 1 \]

\[ y_6 = R\{0.91 + 0.05\} = R\{0.96\} \approx 1 \]

\[ y_7 = R\{0.96 + 0.04\} = R\{1.0\} = 1 \]
Histogram Modification and Equalization

\[ P_d(y_k) \]

\[ 0 \leq y_k \leq 1 \]
Histogram Modification and Equalization

• If input and output range from 0 to L-1 – procedure can be modified as follows:

• **Procedure**: \( x_k = k; \ k=0,\ldots, L-1 = \text{input amplitude levels} \)
  \( y_k; \ k=0,\ldots, L-1 = \text{output amplitude levels} \)
  1. Compute the histogram of the image to be improved.
  2. Normalize histogram;
     ✓ Normalize amplitudes so that the sum of all values is equal to one and you have a pdf, \( p_i(\cdot) \).
  3. Compute
     \[
     y_k = R \left( (L-1) \sum_{l=0}^{k} p_l(x_l) \right)
     \]
     where \( R\{\cdot\} \) - round to next level
  4. Move bins in \( x_k \) to locations in \( y_k \).
  5. Scale \( y_k \) to desired amplitude range (linear mapping).

**Note**: Resulting images have more contrast but appear somewhat unnatural ⇒ better use non-uniform \( h_d(x) \).
**Histogram Modification and Equalization**

- **Example:**

  \[ y_0 = R\{7 p_i(x_0)\} = R\{7(0.14)\} = 1 \]
  \[ y_1 = R\left\{7 \sum_{l=0}^{1} p_i(x_l)\right\} = R\{7(0.14 + 0.25)\} = 3 \]
  \[ y_2 = R\{7(0.39 + 0.2)\} = R\{7(0.59)\} = 5 \]
  \[ y_3 = R\{7(0.59 + 0.15)\} = R\{7(0.74)\} = 6 \]

  \[ y_4 = R\{7(0.74 + 0.1)\} = R\{7(0.84)\} = 6 \]
  \[ y_5 = R\{7(0.84 + 0.07)\} = R\{7(0.91)\} = 7 \]
  \[ y_6 = R\{7(0.91 + 0.05)\} = R\{7(0.96)\} = 7 \]
  \[ y_7 = R\{7(0.96 + 0.04)\} = R\{7(1.0)\} = 7 \]

\[ L = 8 \]

Find \( y_k \) such that \( x_k \) maps into \( y_k \):

\[ y_k = R\left\{L \sum_{i=0}^{k} p_i(x_l)\right\} = R\left\{7 \sum_{i=0}^{k} p_i(x_l)\right\} \]
Histogram Modification and Equalization

\[ P_\alpha(y_k) \]

\[ 0.05 \quad 0.10 \quad 0.15 \quad 0.20 \quad 0.25 \]

\[ 0 \quad 1 \quad 3 \quad 5 \quad 6 \quad 7 \]

\[ y_k; 0 \leq y_k \leq 7 \]
Histogram Modification and Equalization

Original Image

Equalized Image
Histogram Modification and Equalization

• General histogram modification

Compute \( h_i(x_k) \xrightarrow{\text{Normalize to get}} p_i(x_k) \); Let \( F_i(x_k) = \text{input cdf} \)

Let \( h_D(y_k) \xrightarrow{\text{Normalize to get}} p_D(y_k) \); Let \( F_o(y_k) = \text{desired output cdf} \)

\( x_k; k=0,\ldots,L = \text{input amplitude levels} \)

\( y_k; k=0,\ldots,L = \text{output amplitude levels} \)
Histogram Modification and Equalization

- We want to transform $x_k$ with pdf $p_i(x)$ into $y_k$ with pdf $p_D(y_k)$

\[
\begin{align*}
  x_k \overset{\text{with pdf } p_i(x)}{\longrightarrow} & \quad t_k \overset{\text{uniformly distributed}}{\longrightarrow} \quad F^{-1}_i(t_k) \overset{\text{input cumulative distribution function}}{\longrightarrow} y_k \quad \equiv \\
  F^{-1}_o(F_i(x_k)) \overset{\text{output cumulative distribution function}}{\longrightarrow} & \quad y_k
\end{align*}
\]

\[F_i(x_k) = \sum_{l=0}^{k} p_i(x_l) = \text{input cumulative distribution function}\]

\[F_o(y_k) = \sum_{l=0}^{k} p_o(y_l) = \text{output cumulative distribution function}\]

**Note:** $F_o^{-1}(x) = x$ for equalization since cdf linear when $y_k$ uniformly distributed
Histogram Modification and Equalization

✓ **Note:**

\[ t_k = F_i(x_k) \text{ as in histogram equalization} \]

\[ y_k = F_o^{-1}(t_k) \Rightarrow t_k = F_o(y_k) \]

\[ \Rightarrow F_o(y_k) = F_i(x_k) \Rightarrow \text{compute } h_o(y_k) \text{ such that } F_o(y_k) = F_i(x_k) \]
Histogram Modification and Equalization

- **Procedure:**
  1. Compute and normalize amplitude for input histogram $h_i(x_k)$ to get $p_i(x_k)$ (no need to scale the range).
  2. Normalize amplitude of desired histogram $h_o(y_k)$ to get $p_o(y_k)$.
  3. Compute
     \[
     F_i(x_k) = \sum_{l=0}^{k} p_i(x_l); \quad k = 0, \ldots, L - 1
     \]
     \[
     F_o(y_k) = \sum_{l=0}^{k} p_o(y_l); \quad k = 0, \ldots, L - 1
     \]
     $L = \text{number of desired intensity levels}$
  4. For each input level $x_k$; $k=0, \ldots, L-1$
     1. Find output level $y_k$ such that
     \[
     F_o(y_{k-1}) < F_i(x_k) \leq F_o(y_k)
     \]
     2. Output histogram can be simply obtained by assigning bins corresponding to $x_k$ to location $y_k$ $\Rightarrow x_k \rightarrow y_k$
Histogram Modification and Equalization

- **Example:** \( L = 4 \) \( 4 \times 4 \) image \( \Rightarrow \) 16 pixels

\[
F_i(x_k) = \sum_{l=0}^{k} h_i(x_l)
\]

\[
F_i(0) \leq F_o(1) \Rightarrow 0 \rightarrow 1
\]

\[
F_o(1) < F_i(1) = 12 \leq F_o(2) \Rightarrow 1 \rightarrow 2
\]

\[
F_o(1) < F_i(2) = F_o(2) \Rightarrow 2 \rightarrow 2
\]

\[
F_o(2) < F_i(3) = F_o(3) \Rightarrow 3 \rightarrow 3
\]